Systems Infrastructure for Data Science

Web Science Group

Uni Freiburg

WS 2012/13

Lecture II: Indexing

Indexing

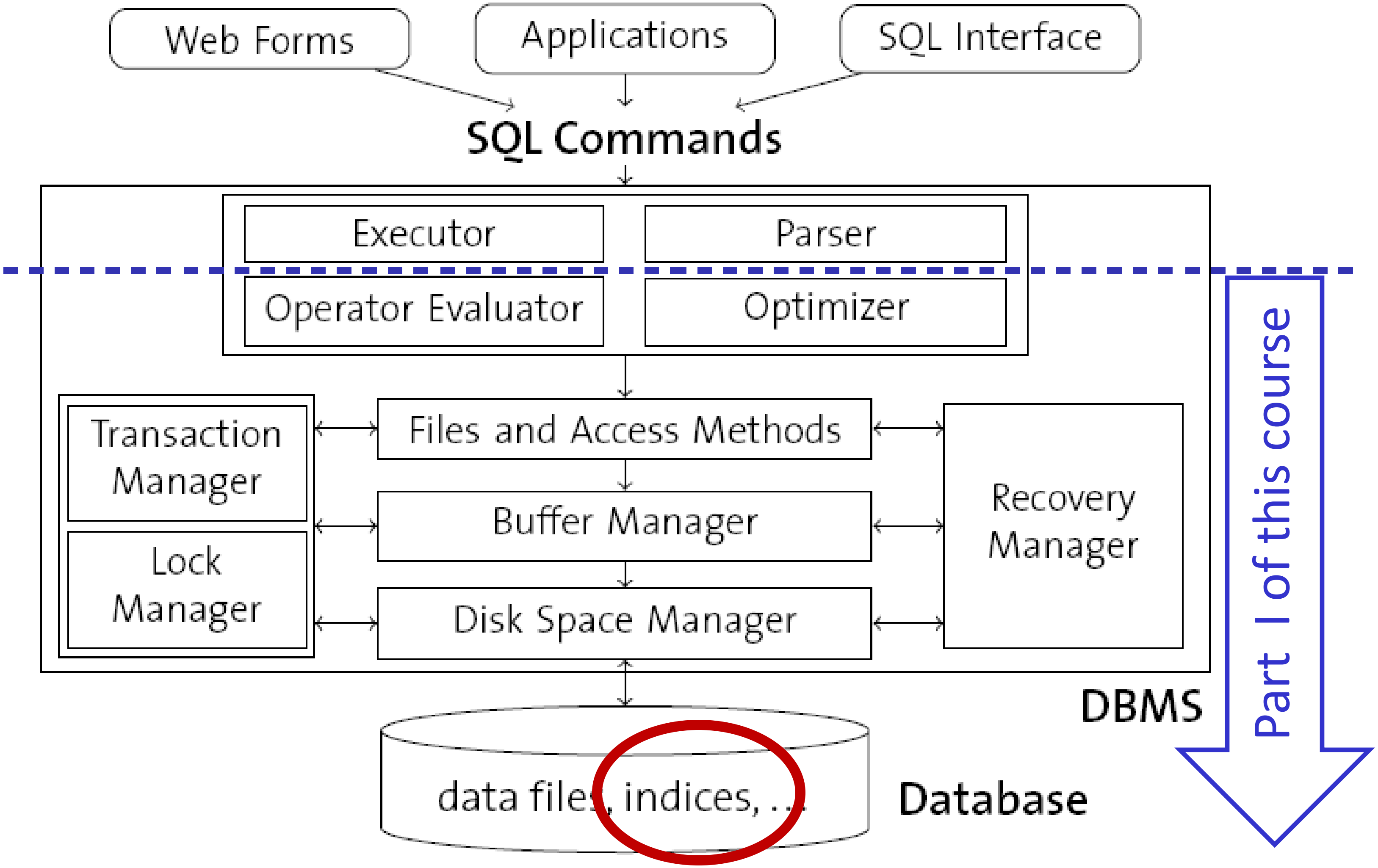


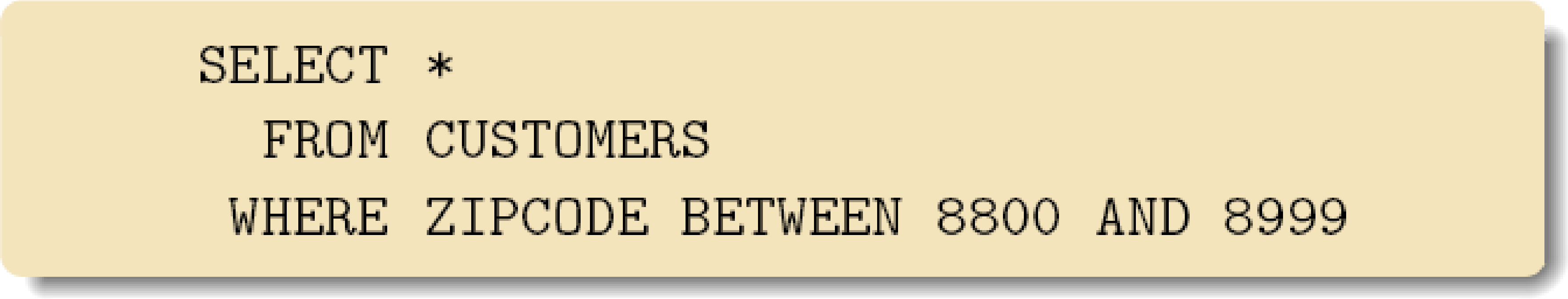
Figure inspired by Ramakrishnan/Gehrke: "Database Management Systems", McGraw-Hill 2003.

Database File Organization and Indexing

- Remember: Database tables are implemented as files of records:
  - A **file** consists of one or more **pages**.
  - Each page contains one or more **records**.
  - Each record corresponds to one **tuple** in a table.
- **File organization:** Method of arranging the records in a file when the file is stored on disk.
- **Indexing:** Building data structures that organize data records on disk in (multiple) ways to optimize search and retrieval operations on them.

File Organization

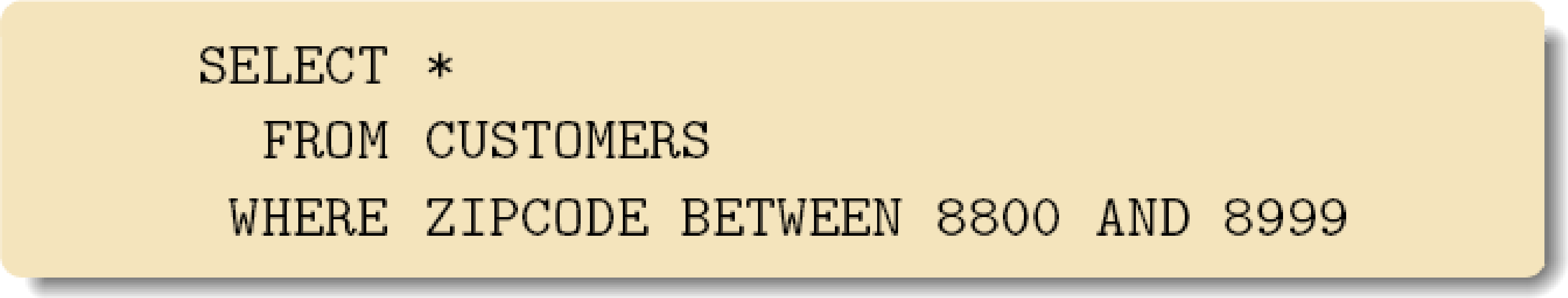
- Given a query such as the following:



```
SELECT *
  FROM CUSTOMERS
 WHERE ZIPCODE BETWEEN 8800 AND 8999
```

- How should we organize the storage of our data files on disk such that we can evaluate this query efficiently?

Heap Files?



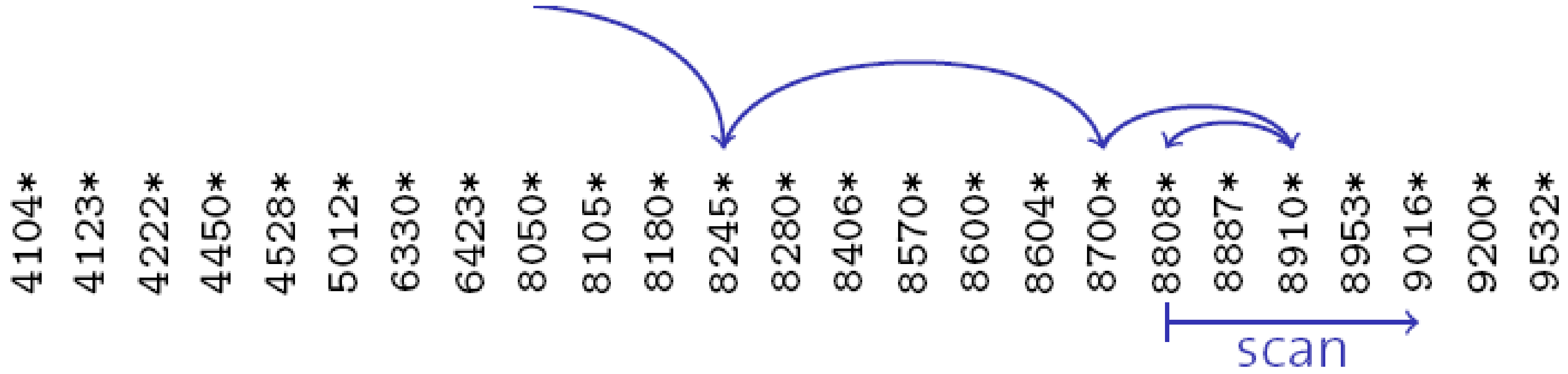
```
SELECT *
  FROM CUSTOMERS
 WHERE ZIPCODE BETWEEN 8800 AND 8999
```

- A heap file stores records in **no particular order**.
- Therefore, CUSTOMER table consists of records that are randomly ordered in terms of their ZIPCODE.
- **The entire file must be scanned**, because the qualifying records could appear anywhere in the file and we don’t know in advance how many such records exist.

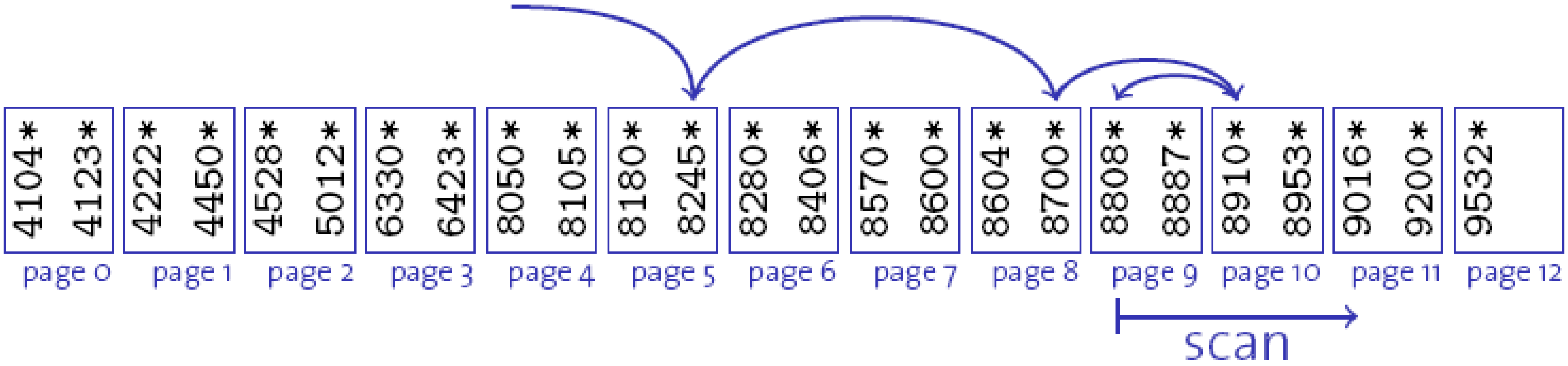
Sorted Files?

```
SELECT *
  FROM CUSTOMERS
 WHERE ZIPCODE BETWEEN 8800 AND 8999
```

- **Sort** the CUSTOMERS table in ZIPCODE order.
- Then use **binary search** to find the first qualifying record, and **scan** further as long as ZIPCODE < 8999.



Are Sorted Files good enough?

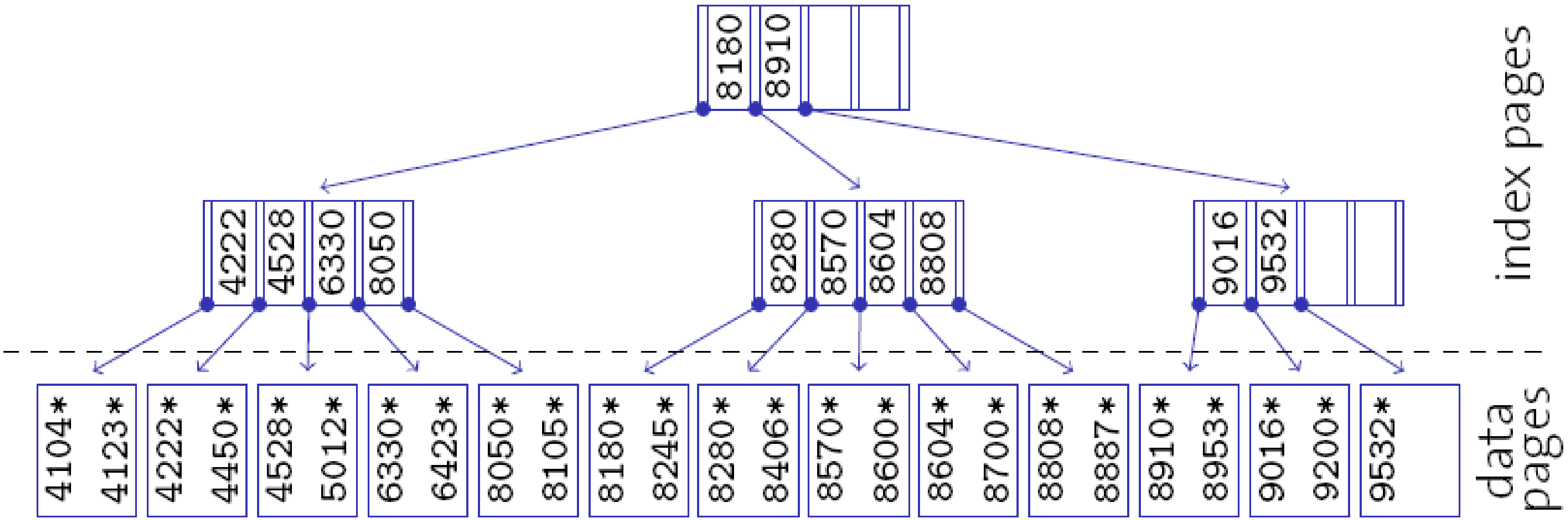


- ✓ Scan phase: We get **sequential access** during this phase.
- ✗ Search phase: We need to read **$\log_2 N$** records during this phase (N: total number of records in the CUSTOMER table).
  - We need to fetch as many pages as are required to access these records.
  - Binary search involves unpredictable jumps that makes prefetching difficult.
- ✗ What about insertions and deletions?

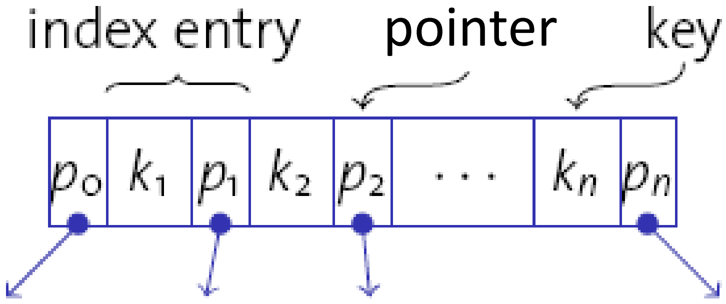
Tree-based Indexing

- Can we reduce the number of pages fetched during the **search phase**?
- Tree-based indexing:
  - Arrange the data entries in sorted order by search key value (e.g., ZIPCODE).
  - Add a hierarchical search data structure on top that directs searches for given key values to the correct page of data entries.
  - Since the index data structure is much smaller than the data file itself, the binary search is expected to fetch a smaller number of pages.
  - Two alternative approaches: **ISAM** and **B^+-tree**.

ISAM: **I**ndexed **S**equential **A**ccess **M**ethod

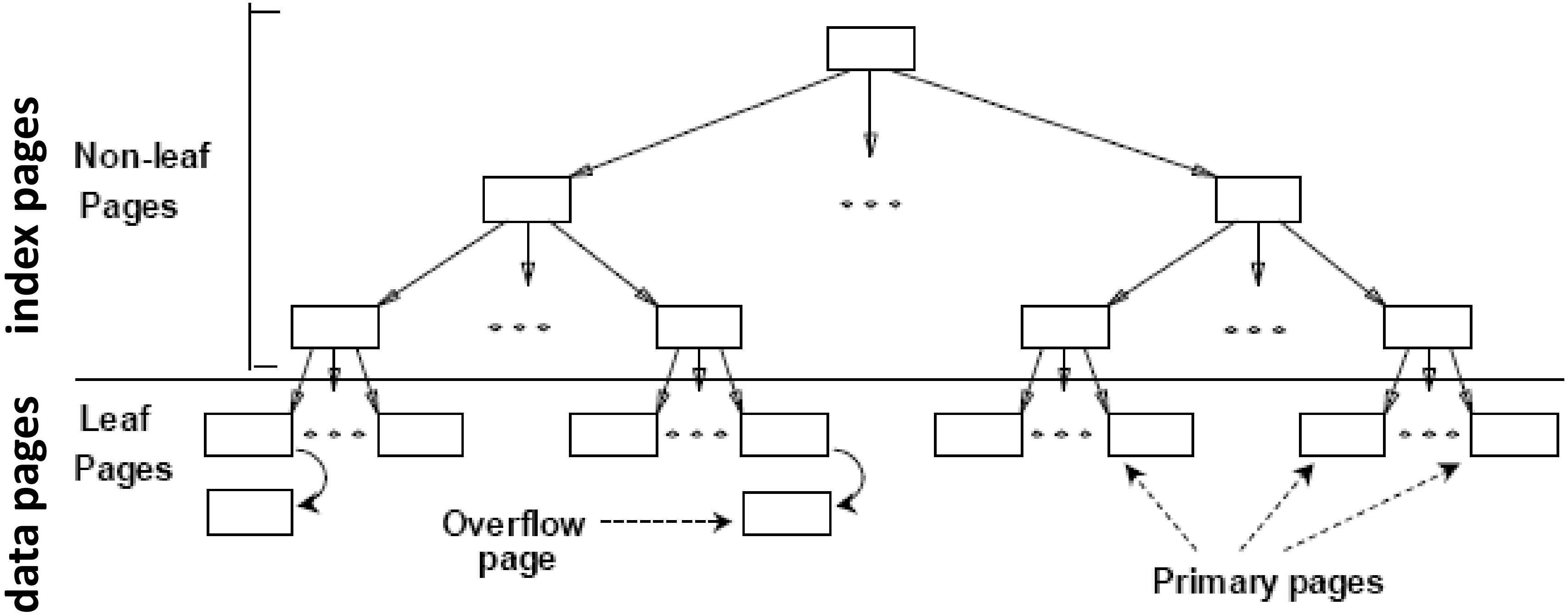


- All nodes are of the size of a page.
  - hundreds of entries per page
  - large fan-out, low depth
- Search cost ~ $\log_{\text{fan-out}} N$
- Key k_i serves as a “separator” for the pages pointed to by p_{i-1} and p_i.



ISAM Index Structure

- **Index pages** stored at non-leaf nodes
- **Data pages** stored at leaf nodes
  - **Primary** data pages & **Overflow** data pages

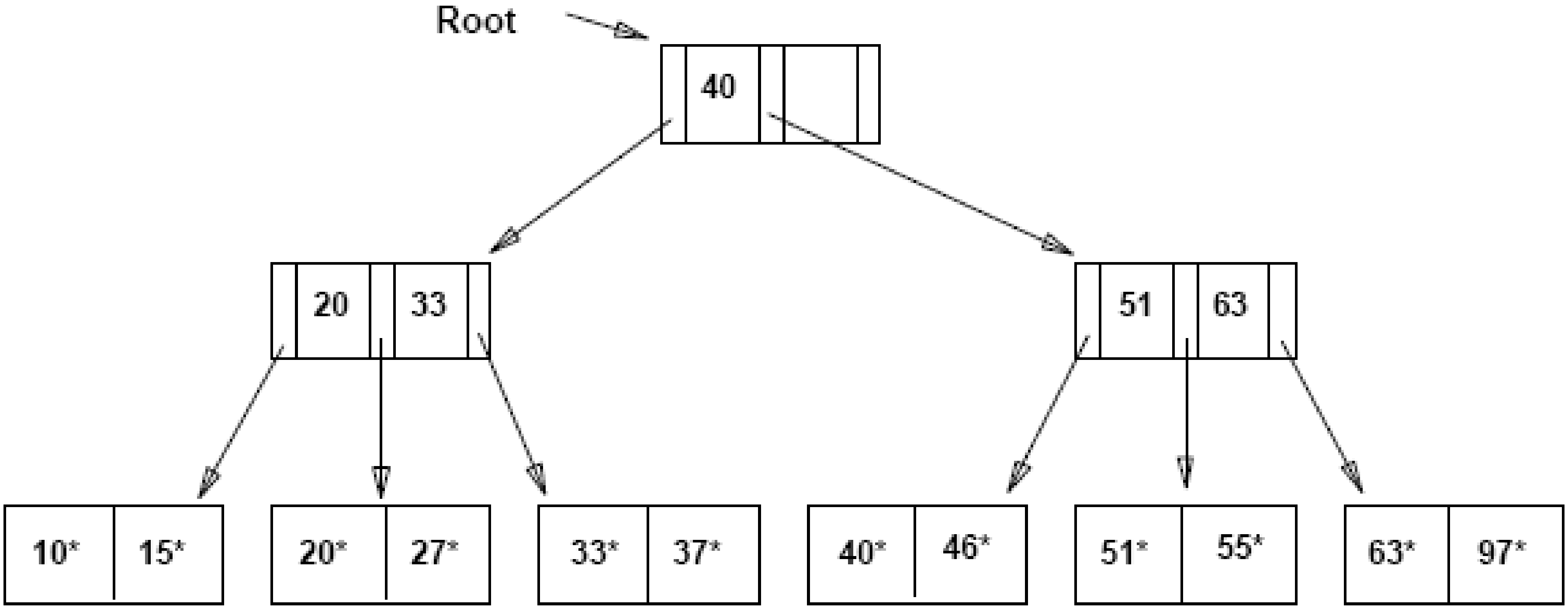


Updates on ISAM Index Structure

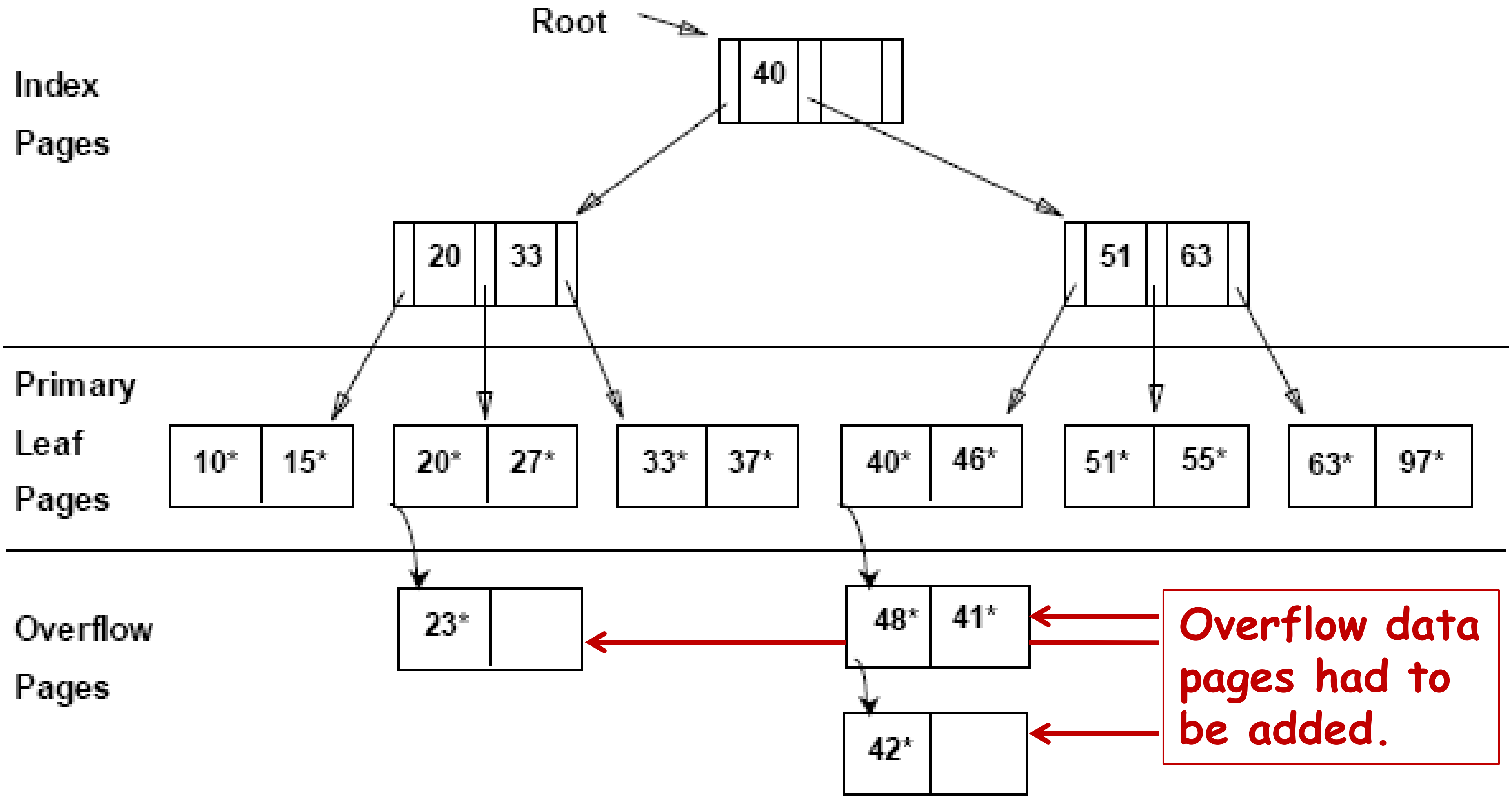
- ISAM index structure is inherently **static**.
  - **Deletion** is not a big problem:
    - Simply remove the record from the corresponding data page.
    - If the removal makes an overflow data page empty, remove that overflow data page.
    - If the removal makes a primary data page empty, keep it as a placeholder for future insertions.
    - Don’t move records from overflow data pages to primary data pages even if the removal creates space for doing so.
  - **Insertion** requires more effort:
    - If there is space in the corresponding primary data page, insert the record there.
    - Otherwise, an **overflow data page** needs to be added.
    - **Note that the overflow pages will violate the sequential order**.
      - ➢ ISAM indexes degrade after some time.

ISAM Example

- Assume: Each node can hold two entries.

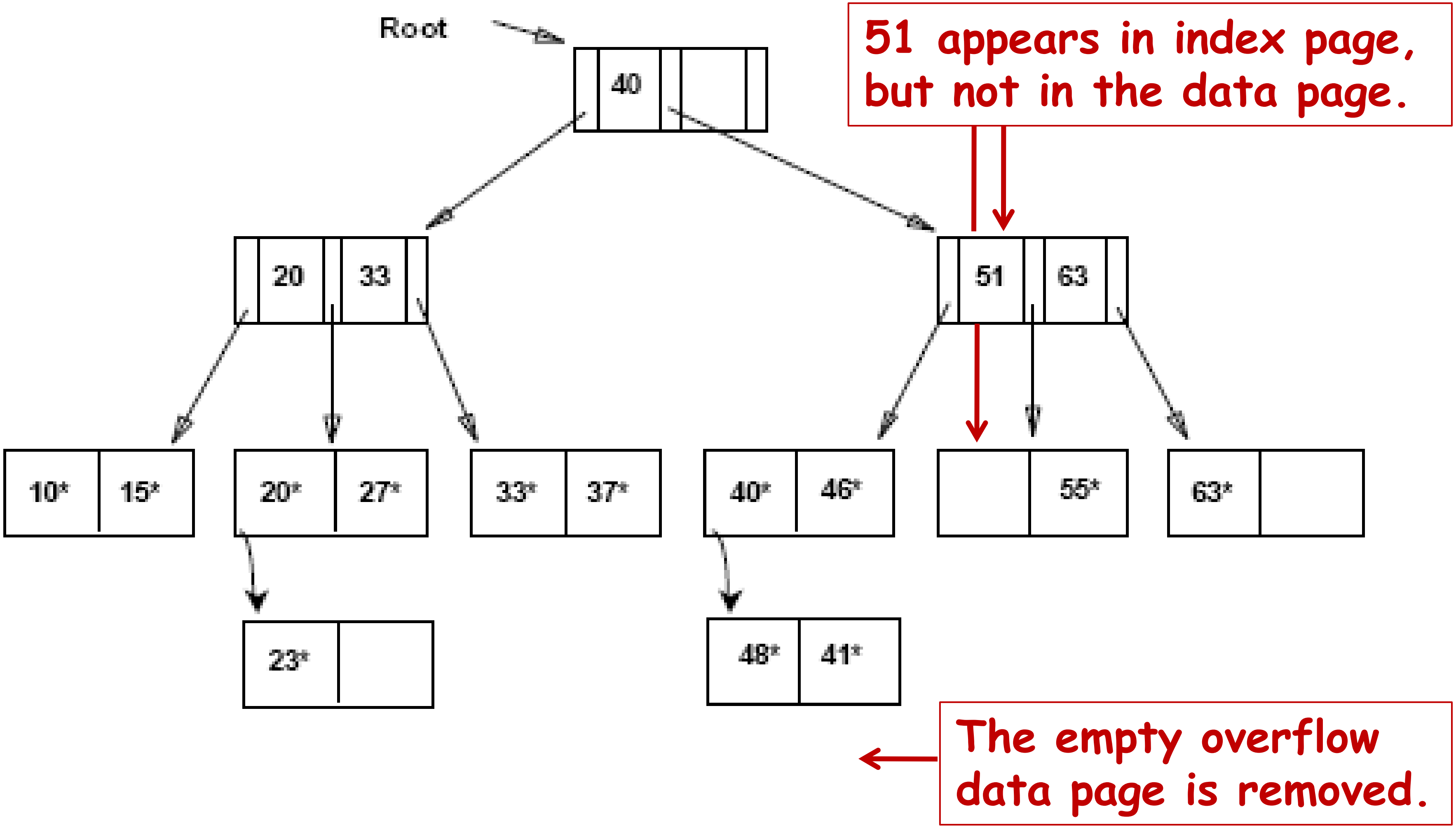


After Inserting 23*, 48*, 41*, 42*



Overflow data pages had to be added.

… Then Deleting 42*, 51*, 97*



51 appears in index page, but not in the data page.

The empty overflow data page is removed.

ISAM: Overflow Pages & Locking

- The non-leaf pages that hold the index data are static; updates affect only the leaf pages.
  - ➢ May lead to **long overflow chains**.
- Leave some **free space** during index creation.
  - ➢ Typically ~ 20% of each page is left free.

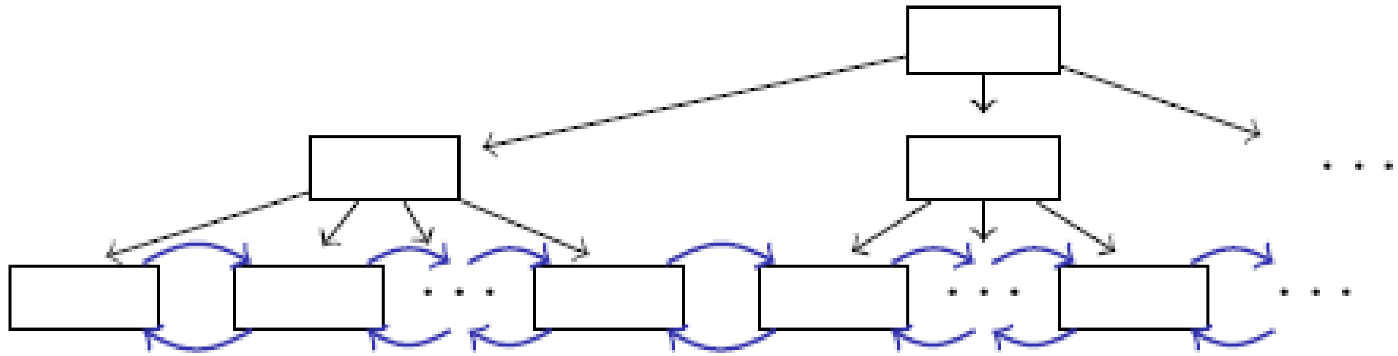
- Since ISAM indexes are static, pages **need not be locked** during index access.
  - – Locking can be a serious bottleneck in dynamic tree indexes (particularly near the root node).
- ISAM may be the index of choice for relatively static data.

B^+-trees: A Dynamic Index Structure

- The **B^+-tree** is derived from the ISAM index, but is **fully dynamic** with respect to updates.
  - **No overflow chains**; B^+-trees remain **balanced** at all times.
  - Gracefully adjusts to insertions and deletions.
  - **Minimum occupancy** for all B^+-tree nodes (except the root): 50% (typically: 67 %).
  - Original version:
    - **B-tree**: R. Bayer and E. M. McCreight, “Organization and Maintenance of Large Ordered Indexes”, Acta Informatica, vol. 1, no. 3, September 1972.

B+-trees: Basics

- B+-trees look like ISAM indexes, where
  - leaf nodes are, generally, **not in sequential order** on disk
  - leaves are typically connected to form a **doubly-linked list**
  - leaves may contain **actual data** (like the ISAM index) or just **references** to data pages (e.g., record ids (rids))
    - We will assume the latter case, since it is the more common one.
  - each B+-tree node contains between *d* and *2d* entries (*d* is the **order** of the B+-tree; the root is the only exception).

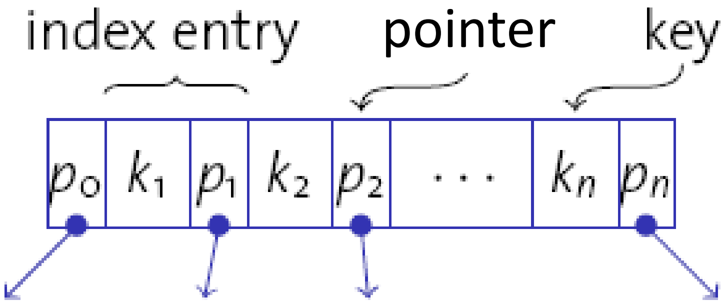


Searching a B^+-tree

```
Function: search ($k$)
return tree_search ($k$, $root$);
```

```
Function: tree_search ($k$, $node$)
if $node$ is a leaf then
    return $node$;
switch $k$ do
    case $k < k_0$
        return tree_search ($k$, $p_0$);
    case $k_i \leq k < k_{i+1}$
        return tree_search ($k$, $p_i$);
    case $k_{2d} \leq k$
        return tree_search ($k$, $p_{2d}$);
```

- Function *search (k)* returns a pointer to the leaf node that contains potential hits for search key *k*.
- Node page layout:



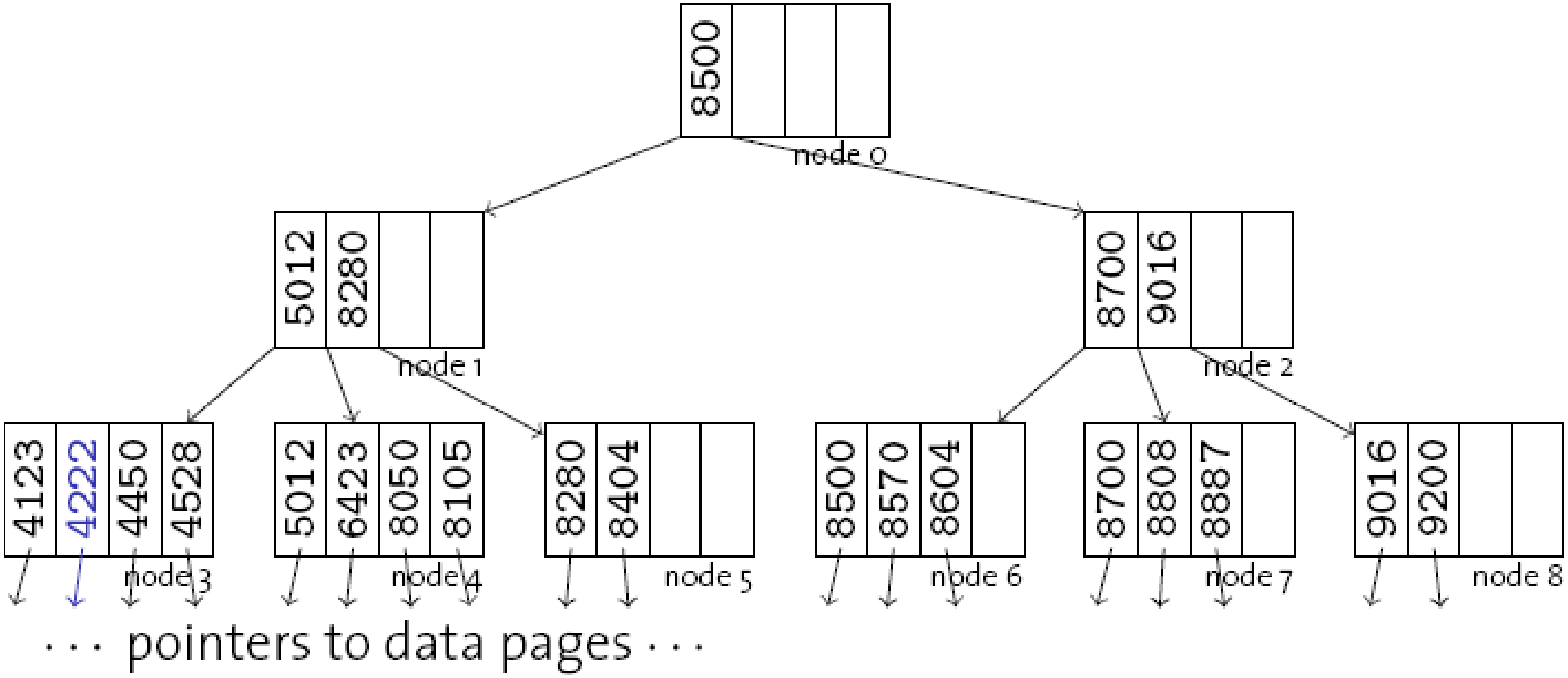
Insertion to a B^+-tree: Overview

- The B^+-tree needs to remain **balanced** after every update (i.e., every root-to-leaf path must be of the same length).
  - ➢ We cannot create overflow pages.

- Sketch of the insertion procedure for entry <*k, p*> (key value *k* pointing to data page *p*):
  1. **Find leaf page** *n* where we would expect the entry for *k*.
  2. If *n* has **enough space** to hold the new entry (i.e., at most *2d-1* entries in *n*), **simply insert** <*k, p*> into *n*.
  3. Otherwise, node *n* must be **split** into *n* and *n'*, and a new **separator** has to be inserted into the parent of *n*.

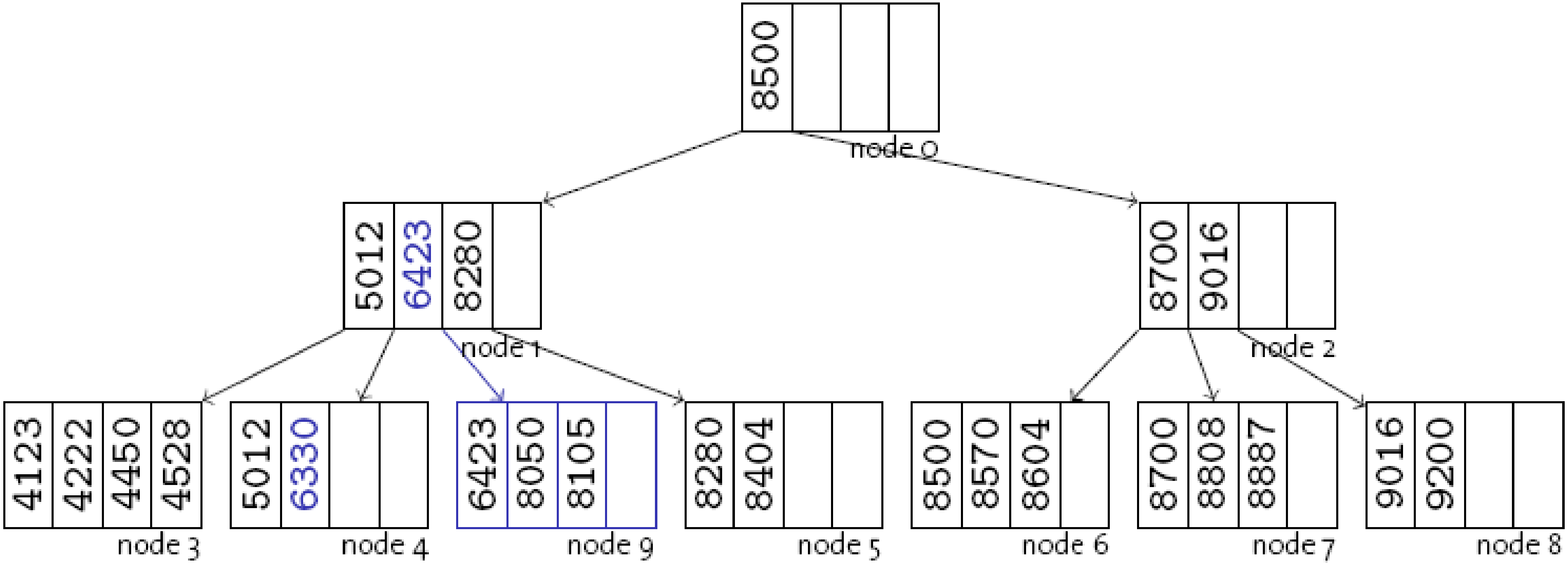
     Splitting happens recursively and may eventually lead to a split of the root node (increasing the height of the tree).

Insertion to a B$^+$-tree: Example

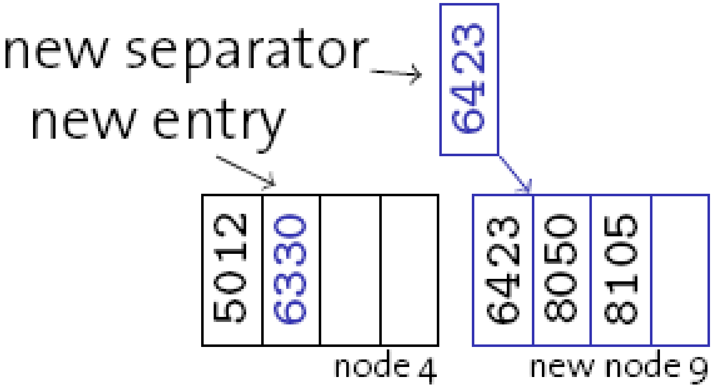


- Insert new entry with key **4222**.
  - Enough space in node 3, simply insert without split.
  - Keep entries **sorted within nodes**.

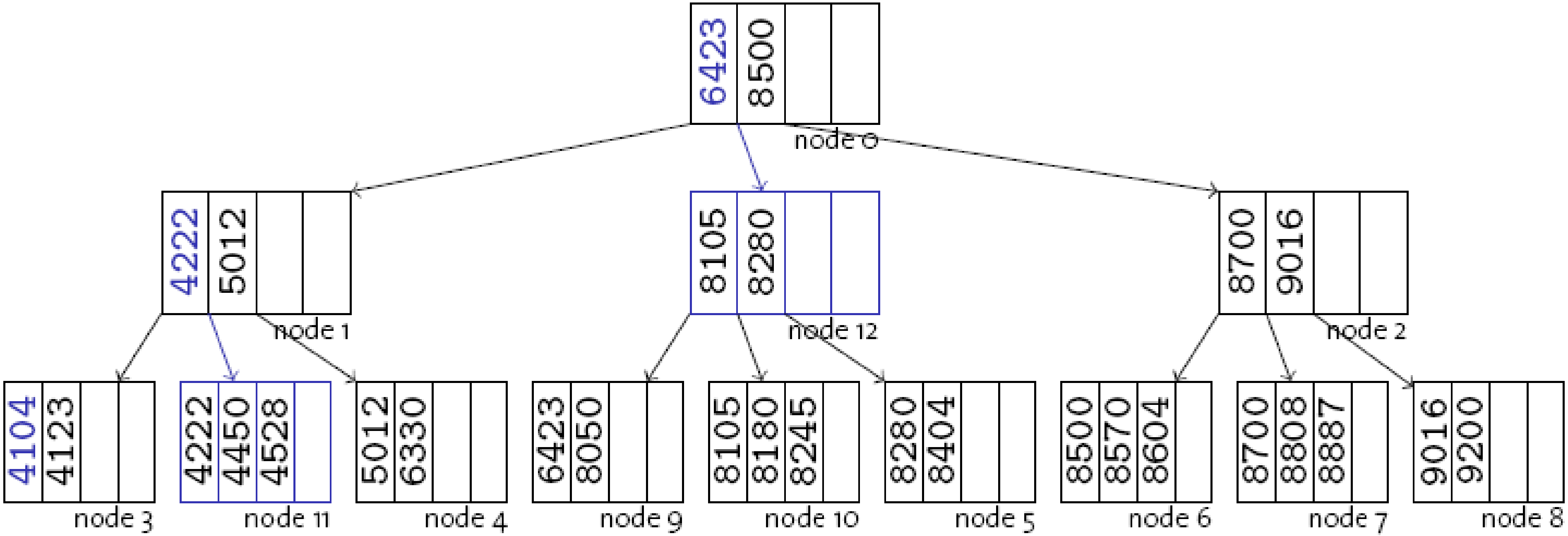
Insertion to a B⁺-tree: Example



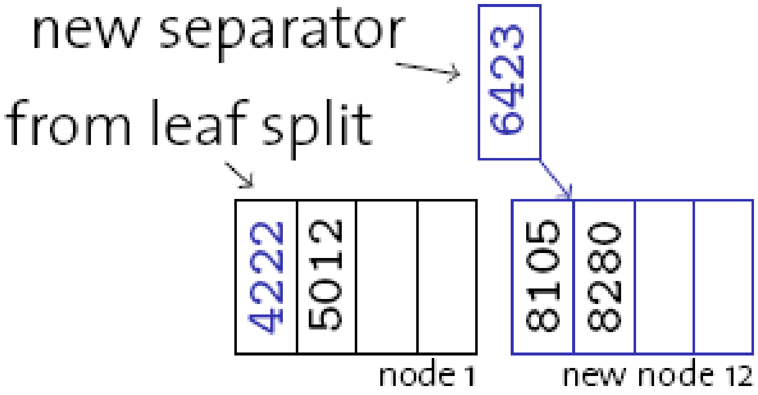
- Insert key **`6330`**.
  - Must **split** node 4.
  - **New separator** goes into node 1 (including pointer to new page).



Insertion to a B^+-tree: Example



- After **8180**, **8245**, insert key **4104**.
  - Must **split** node 3.
  - Node 1 **overflows** => split it!
  - **New separator** goes into root.
- Note: Unlike during leaf split, separator key does not remain in inner node.



Insertion to a B$^+$-tree: Root Node Split

- Splitting starts at the leaf level and continues upward as long as index nodes are fully occupied.
- Eventually, this can lead to a split of the root node:
  - Split like any other inner node.
  - Use the separator to create a new root.
- The root node is the only node that may have an occupancy of less than 50 %.
- **This is the only situation where the tree height increases.**

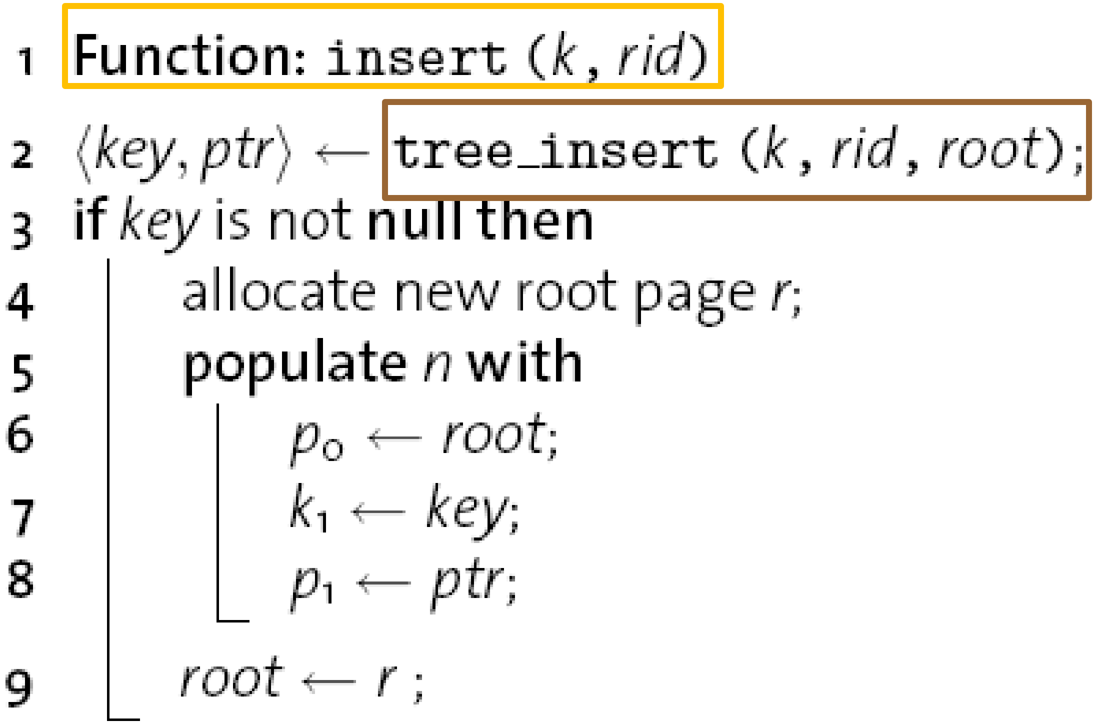
Insertion Algorithm

```
Function: tree_insert ($k$, $rid$, $node$)
if $node$ is a leaf then
    return leaf_insert ($k$, $rid$, $node$);
else
    switch $k$ do
        case $k < k_0$
            $\langle sep, ptr \rangle \leftarrow$ tree_insert ($k$, $rid$, $p_0$);
        case $k_i \leq k < k_{i+1}$
            $\langle sep, ptr \rangle \leftarrow$ tree_insert ($k$, $rid$, $p_i$);
        case $k_{2d} \leq k$
            $\langle sep, ptr \rangle \leftarrow$ tree_insert ($k$, $rid$, $p_{2d}$);
    if $sep$ is null then
        return $\langle$null, null$\rangle$;
    else
        return split ($sep$, $ptr$, $node$);
```

} see tree_search ()

```
Function: leaf_insert ($k$, $rid$, $node$)
if another entry fits into node then
    insert $\langle k, rid \rangle$ into node ;
    return $\langle$null, null$\rangle$;
else
    allocate new leaf page $p$ ;
    take $\{\langle k_1^+, p_1^+ \rangle, \ldots, \langle k_{2d+1}^+, p_{2d+1}^+ \rangle\} :=$ entries from node $\cup \{\langle k, ptr \rangle\}$
        leave entries $\langle k_1^+, p_1^+ \rangle, \ldots, \langle k_d^+, p_d^+ \rangle$ in node ;
        move entries $\langle k_{d+1}^+, p_{d+1}^+ \rangle, \ldots, \langle k_{\cancel{2d}}^+, p_{\cancel{2d}}^+ \rangle$ to $p$ ;
                                                   2d+1      2d+1
    return $\langle k_{d+1}^+, p \rangle$;
```

```
Function: split ($k$, $ptr$, $node$)
if another entry fits into node then
    insert $\langle k, ptr \rangle$ into node ;
    return $\langle$null, null$\rangle$;
else
    allocate new leaf page $p$ ;
    take $\{\langle k_1^+, p_1^+ \rangle, \ldots, \langle k_{2d+1}^+, p_{2d+1}^+ \rangle\} :=$ entries from node $\cup \{\langle k, ptr \rangle\}$
        leave entries $\langle k_1^+, p_1^+ \rangle, \ldots, \langle k_d^+, p_d^+ \rangle$ in node ;
        move entries $\langle k_{\cancel{d+2}}^+, p_{d+1}^+ \rangle, \ldots, \langle k_{\cancel{2d}}^+, p_{\cancel{2d}}^+ \rangle$ to $p$ ;
                         d+1
        ~~set $p_0 \leftarrow p_{d+1}^+$ in node;~~
                                                   2d+1      2d+1
    return $\langle k_{d+1}^+, p \rangle$;
```

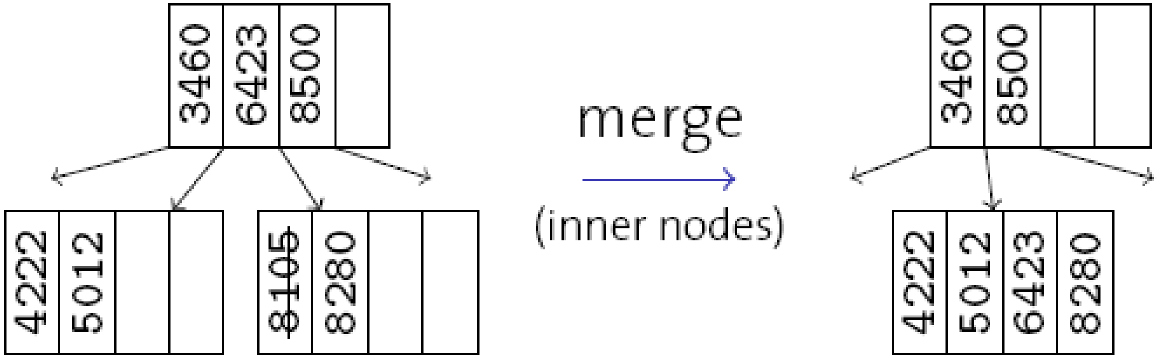


```
Function: insert ($k$, $rid$)
$\langle key, ptr \rangle \leftarrow$ tree_insert ($k$, $rid$, $root$);
if $key$ is not null then
    allocate new root page $r$;
    populate $n$ with
        $p_0 \leftarrow root$;
        $k_1 \leftarrow key$;
        $p_1 \leftarrow ptr$;
    $root \leftarrow r$;
```

- *insert (k, rid)* is called from outside.
- Note how leaf node entries point to rids, while inner nodes contain pointers to other B$^+$-tree nodes.

Deletion from a B^+-tree

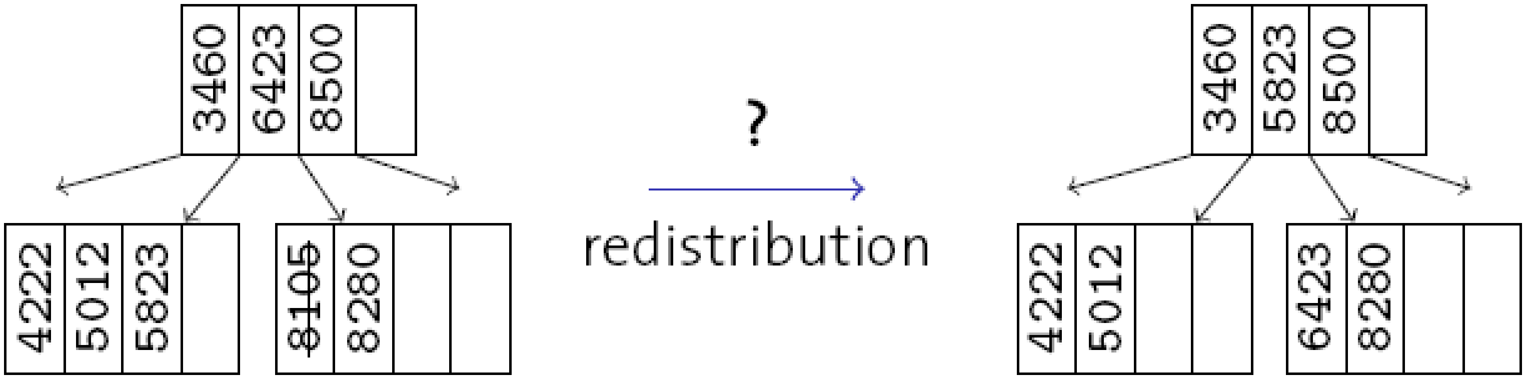
- If a node is sufficiently full (i.e., contains at least *d+1* entries), we may simply remove the entry from the node.
  - Note: Afterwards, **inner nodes** may contain keys that no longer exist in the database. This is perfectly legal.
- Merge nodes in case of an underflow (i.e., “undo” a split):



- “Pull” separator (i.e., key **6423**) into merged node.

Deletion from a B$^+$-tree

- It is not that easy:



- Merging only works if two neighboring nodes were 50% full.
- Otherwise, we have to **re-distribute**:
  - “rotate” entry through parent

B$^+$-trees in Real Systems

- Actual systems often avoid the cost of merging and/or redistribution, but relax the minimum occupancy rule.
- **Example: IBM DB2 UDB**
  - The “`MINPCTUSED`” parameter controls when the system should try a leaf node merge (“on-line index reorganization”).
  - This is particularly easy because of the pointers between adjacent leaf nodes.
  - Inner nodes are never merged (need to do a full table reorganization for that).
- To improve concurrency, systems sometimes only mark index entries as deleted and physically remove them later (e.g., IBM DB2 UDB “type-2 indexes”).

What is stored inside the leaves?

- Basically there are three alternatives:

  1. The full data entry *k**. Such an index is inherently **clustered** (e.g., ISAM).

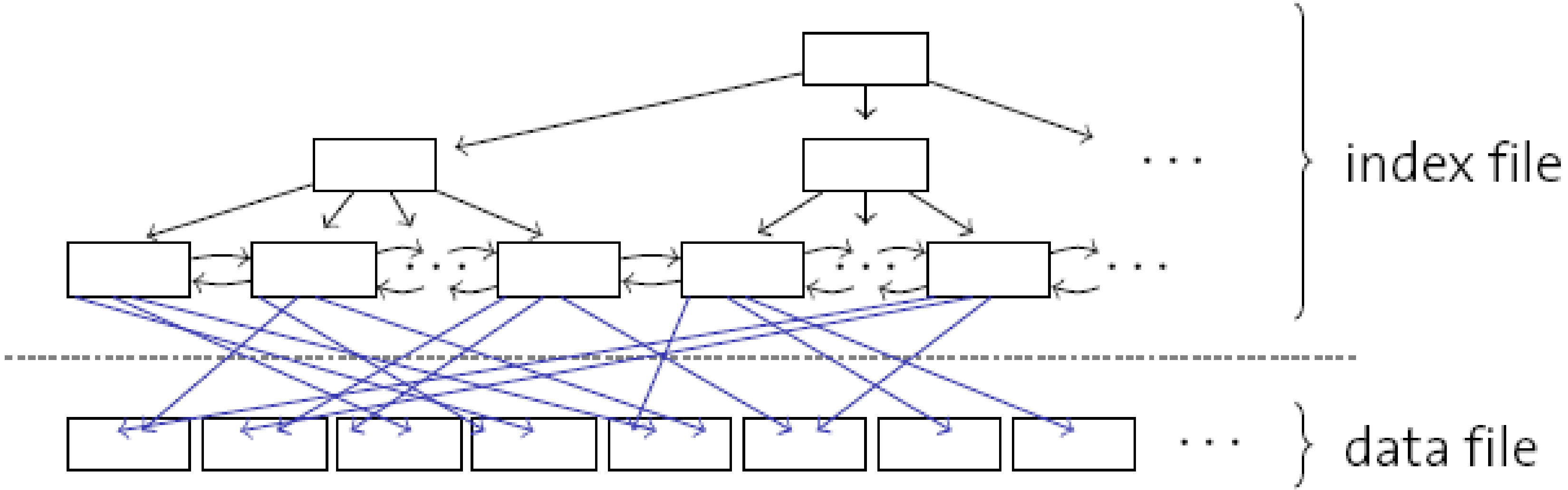
  2. A *<k, rid>* pair, where *rid* is the record id of the data entry.

  3. A *<k, {*rid_1*,* rid_2*, …}>* pair, where the items in the rid list rid_i are record ids of data entries with search key value *k*.

- 2 and 3 are reasons why we want record ids to be stable.
- 2 seems to be the most common one.

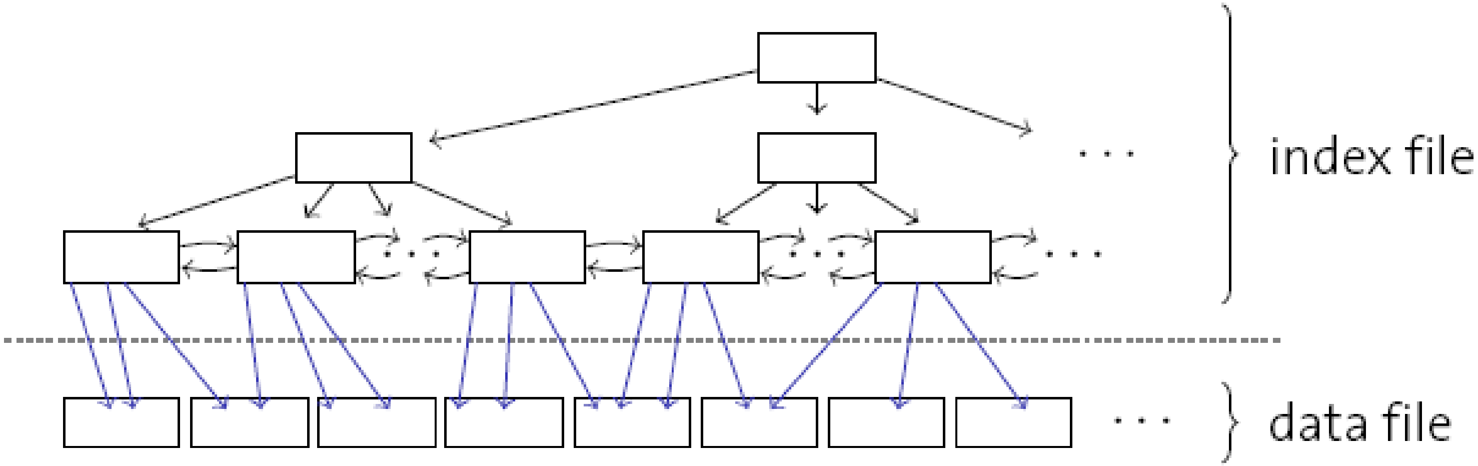
B^+-trees and Sorting

- A typical situation according to alternative 2 looks as follows:



Clustered B^+-trees

- If the data file was sorted, the scenario would look different:



- We call such an index a **clustered index**.
  - Scanning the index now leads to **sequential access**.
  - This is particularly good for **range queries**.

Index-organized Tables

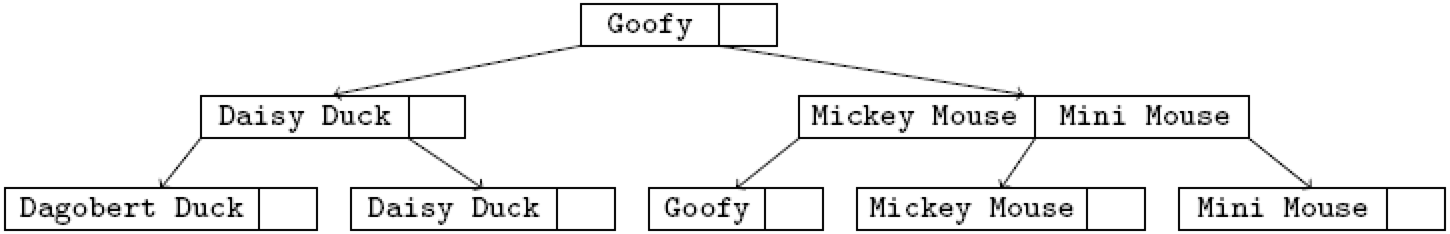
- Alternative 1 is a special case of a clustered index.
  - index file = data file
  - Such a file is often called an **index-organized table**.

- Example: Oracle 8i

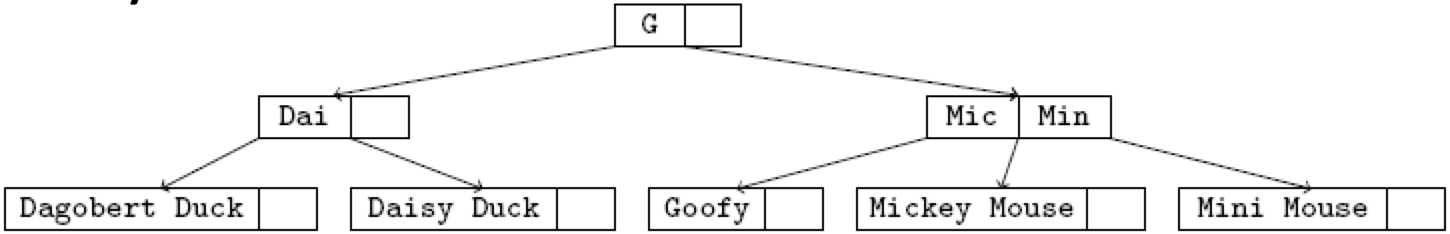
```
CREATE TABLE(...
             ...,
             PRIMARY KEY(...))
ORGANIZATION INDEX;
```

Key Compression: Suffix Truncation

- B$^+$-tree fan-out is proportional to the number of index entries per page, i.e., inversely proportional to the key size.
  - ➢ Reduce key size, particularly for **variable-length strings**.



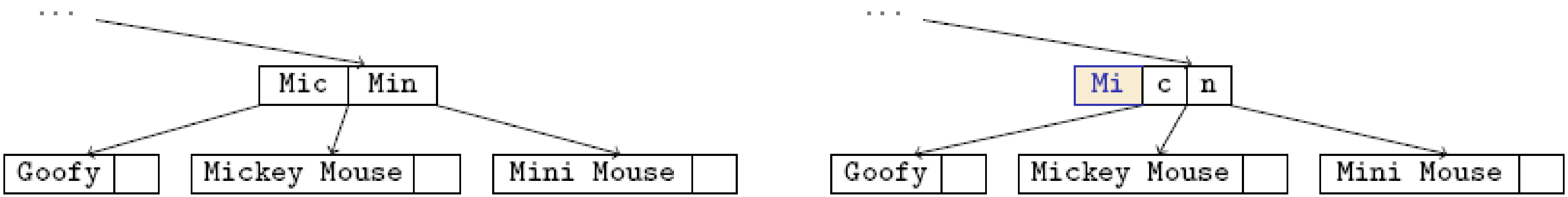
- **Suffix truncation:** Make separator keys only as long as necessary:



- Note that separators need not be actual data values.

Key Compression: Prefix Truncation

- Keys within a node often share a **common prefix.**



- **Prefix truncation:**
  - Store common prefix **only once** (e.g., as “k_0”).
  - Keys have become highly discriminative now.

R. Bayer, K. Unterauer, “Prefix B-Trees”, ACM TODS 2(1), March 1977.

B. Bhattacharjee et al., “Efficient Index Compression in DB2 LUW”, VLDB’09.

Composite Keys

- B$^+$-trees can in theory be used to index everything with a defined **total order** such as:
  - integers, strings, dates, etc., and
  - concatenations thereof (based on lexicographical order)
- Example: In most SQL dialects:

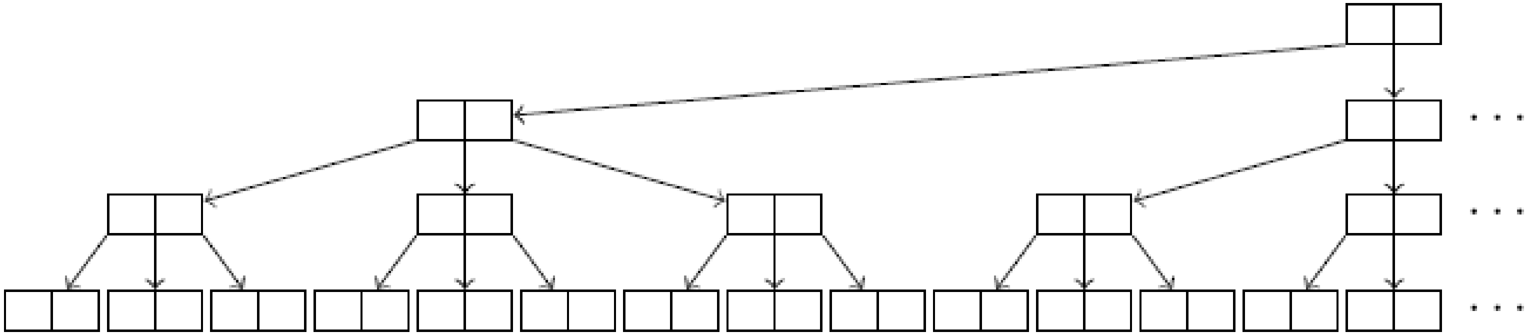
```
CREATE INDEX ON TABLE CUSTOMERS(LASTNAME, FIRSTNAME);
```

- A useful application are, e.g., **partitioned B-trees**:
  - Leading index attributes effectively **partition** the resulting B$^+$-tree.

G. Graefe, “Sorting and Indexing with Partitioned B-Trees”, CIDR’03.

Bulk-Loading B^+-trees

- Building a B+-tree is particularly easy when the input is sorted.



- Build B+-tree **bottom-up** and **left-to-right**.
- Create a parent for every *2d+1* un-parented nodes.
  - Actual implementations typically leave some space for future updates (e.g., DB2’s “**PCTFREE**” parameter).

Stars, Pluses, …

- In the foregoing we described the **B$^+$-tree**.
- Bayer and McCreight originally proposed the **B-tree**:
  - Inner nodes contain data entries, too.
- There is also a **B*-tree**:
  - Keep non-root nodes at least 2/3 full (instead of 1/2).
  - Need to **redistribute on inserts** to achieve this

    => Whenever two nodes are full, split them into three.
- Most people say “B-tree” and mean any of these variations. Real systems typically implement B$^+$-trees.
- “B-trees” are also used outside the database domain, e.g., in modern **file systems** (ReiserFS, HFS, NTFS, ...).

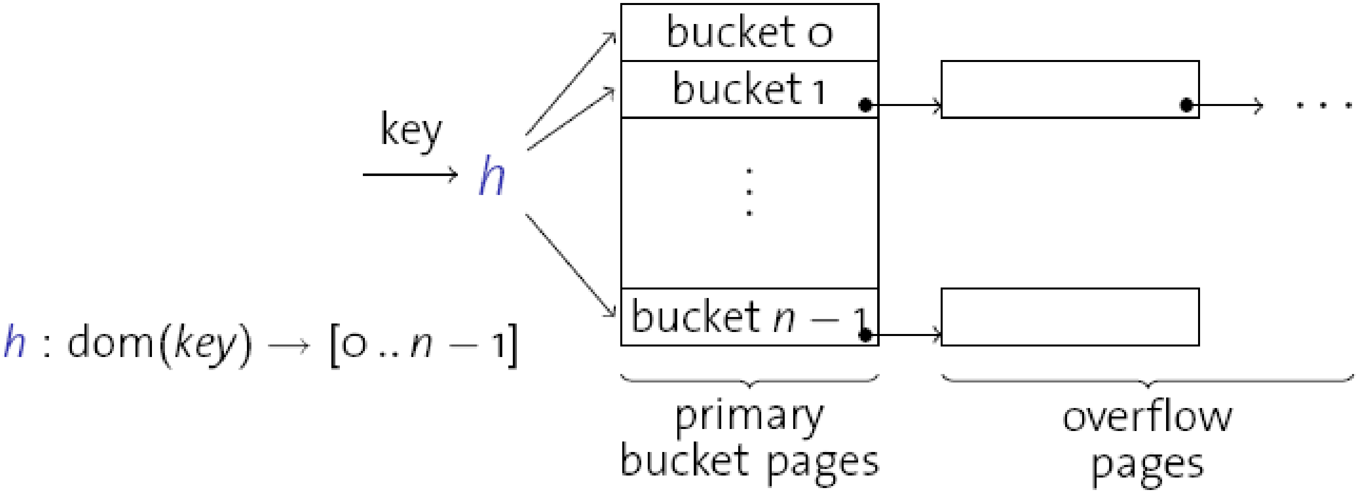
Hash-based Indexing

- B^+-trees are by far the predominant type of indices in databases. An alternative is hash-based indexing.

- Hash indexes can only be used to answer **equality selection queries** (not range selection queries).

- Like in tree-based indexing, static and dynamic hashing techniques exist; their trade-offs are similar to ISAM vs. B^+-trees.

Hash-based Indexing



- Records in a file are grouped into **buckets**.
- A bucket consists of a **primary page** and possibly **overflow pages** linked in a chain.
- **Hash function**:
  - Given a the search key of a record, returns the corresponding bucket number that contains that record.
  - Then we search the record within that bucket.

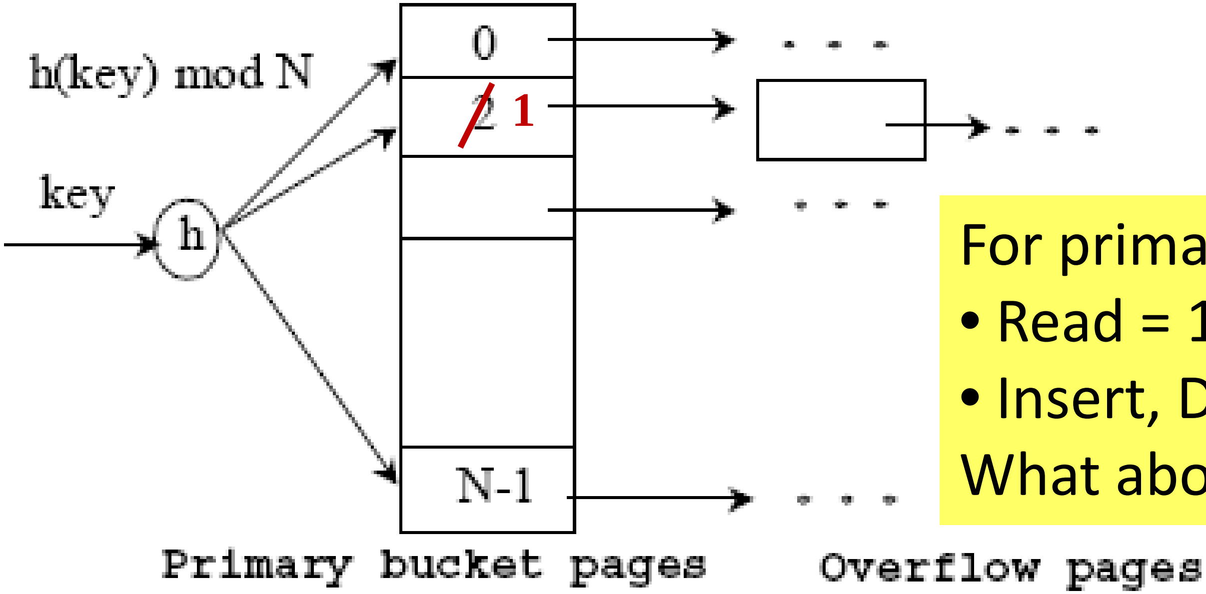
Hash Function

- A good hash function distributes values in the domain of the search key uniformly over the collection of buckets.

- Given *N* buckets 0 .. *N-1*, *h(value) = (a*value + b)* works well.
  - *h(value) mod N* gives the bucket number.
  - *a* and *b* are constants to be tuned.

Static Hashing

- Number of primary pages is fixed.
- Primary pages are allocated sequentially and are never de-allocated. Use overflow pages if need more pages.
- *h(k) mod N* gives the bucket to which the data entry with search key *k* belongs. (*N*: number of buckets)



For primary pages:

- Read = 1 disk I/O
- Insert, Delete = 2 disk I/Os

What about the overflow pages?

Problems with Static Hashing

- Number of buckets *n* is fixed.
  - How to choose *n*?
  - Many deletions => space is wasted
  - Many insertions => long overflow chains that degrade search performance
- Static hashing has similar problems and advantages as in ISAM.
- Rehashing solution:
  - Periodically rehash the whole file to restore the ideal (i.e., no overflow chains and 80% occupancy)
  - Takes long and makes the index unusable during rehashing.

Dynamic Hashing

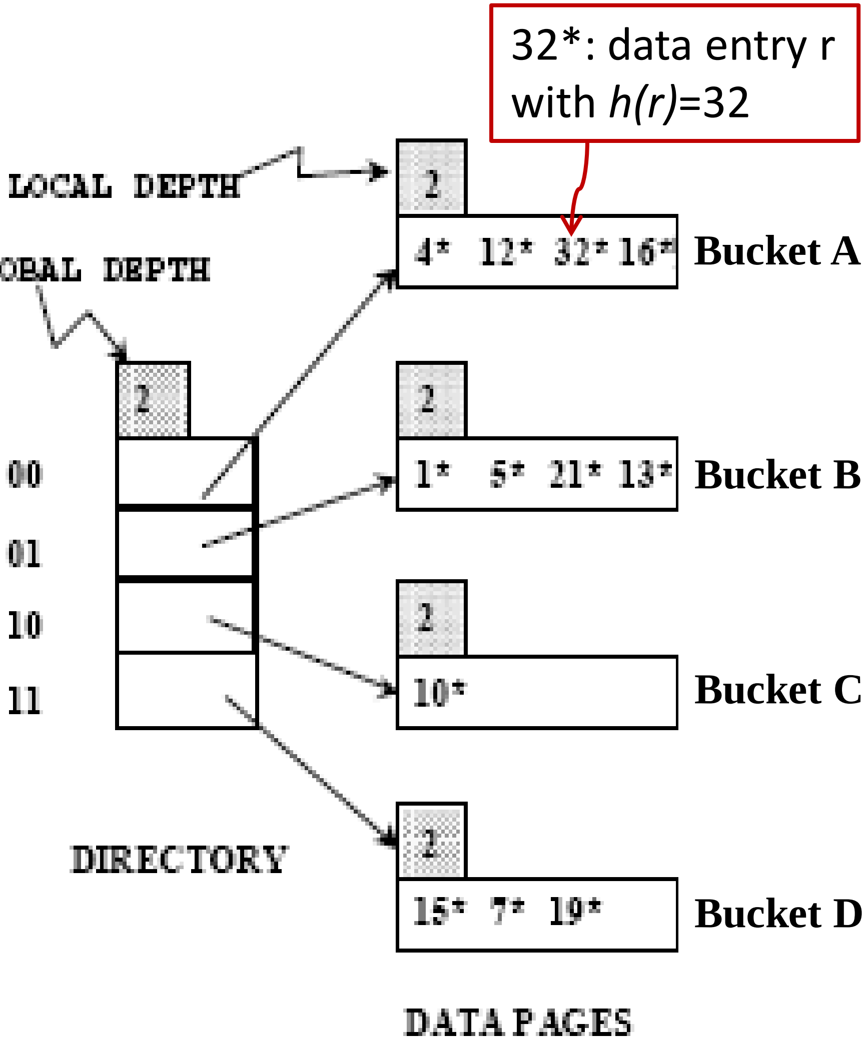
- To deal with the problems of static hashing, database systems use **dynamic hashing** techniques:
  - **Extendible hashing**
  - **Linear hashing**
- Note that: Few real systems support true hash indexes (such as PostgreSQL).
- More popular uses of hashing are:
  - support for B^+-trees over hash values (e.g., SQL Server)
  - the use of hashing during query processing => **hash join**

Extendible Hashing: The Idea

- Overflows occur when bucket (primary page) becomes full. Why not re-organize the file by doubling the number of buckets?
  - Reading and writing all pages is expensive!
- **Idea:** Use a directory of pointers to buckets; double the number of buckets by doubling the directory and splitting just the bucket that overflowed.
  - Directory is much smaller than file, so doubling it is much cheaper. Only one page of data entries is split.
  - **No overflow pages!**
  - Trick lies in how the hash function is adjusted.

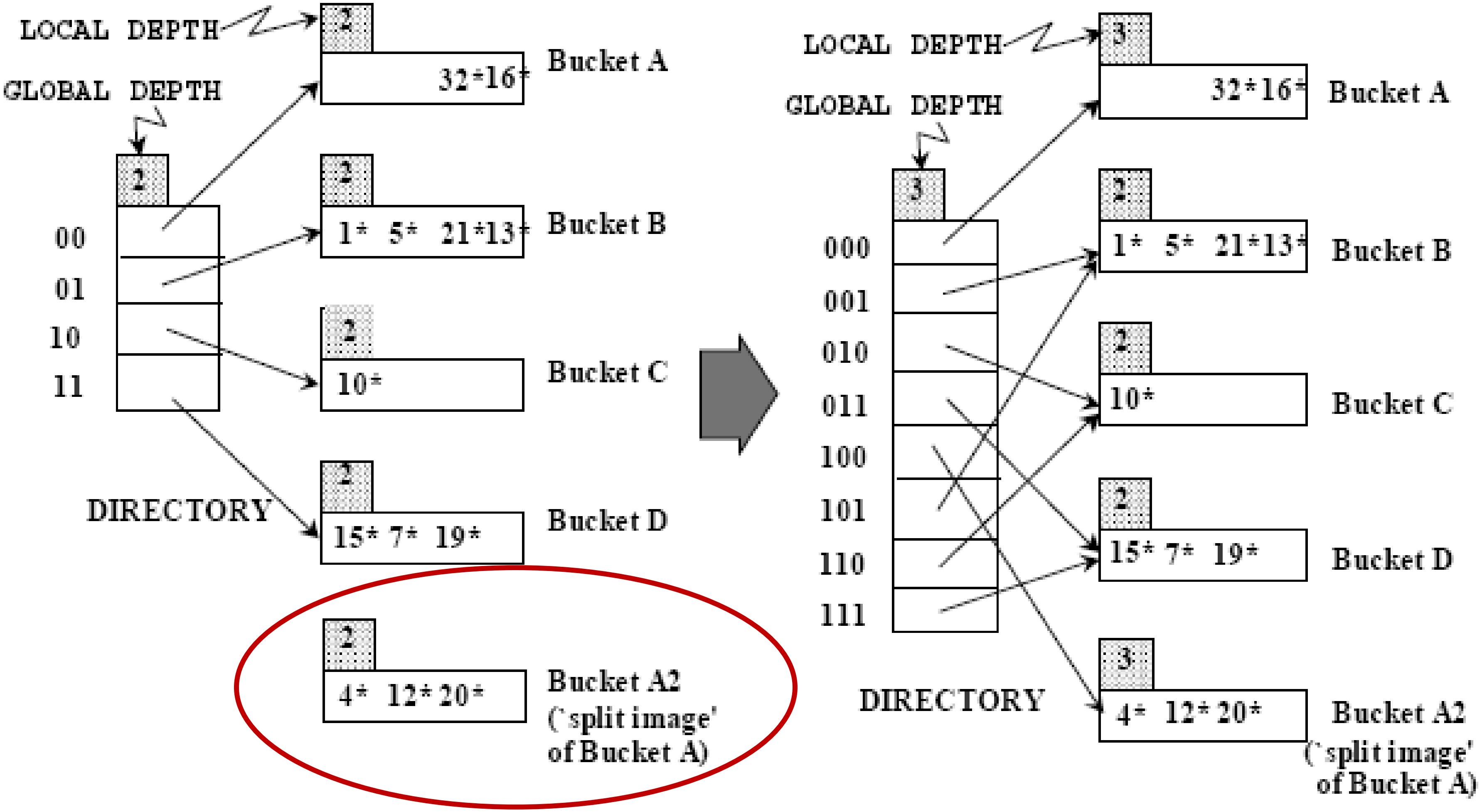
Extendible Hashing: An Example

- The directory is an array of size 4.
- Search:
  - To find the bucket for search key *r*, take the last "global depth" number of bits of *h(r)*:
  - *h(r)* = 5 = binary 1**01** => The data entry for *r* is in the bucket pointed to by **01**.
- Insertion:
  - If the bucket is full, **split** it.
  - If "necessary", **double** the directory.


32*: data entry r with *h(r)*=32

Extendible Hashing: Directory Doubling

Insert 20*: *h(r)* = 20 = binary 101**00**

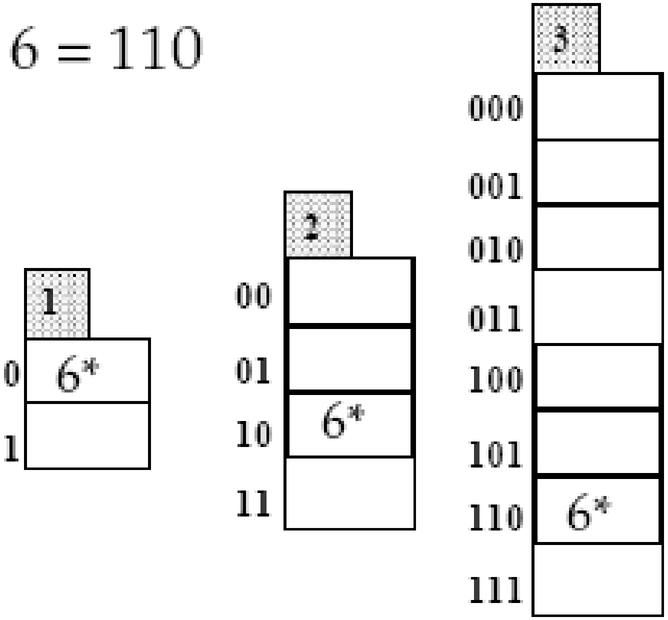


Extendible Hashing: Directory Doubling

- 20 = binary 10100. The last 2 bits (00) tell us that *r* belongs in bucket **A or A2**. The last 3 bits are needed to tell which.
  - **Global depth** of directory = maximum number of bits needed to tell which bucket an entry belongs to.
  - **Local depth** of a bucket = number of bits used to determine if an entry belongs to a given bucket.
- When does a bucket split cause directory doubling?
  - Before the insertion and split, **local depth = global depth**.
  - After the insertion and split, **local depth > global depth**.
  - Directory is doubled by copying it over and fixing the pointer to the split image page.
  - After the doubling, **global depth = local depth**.

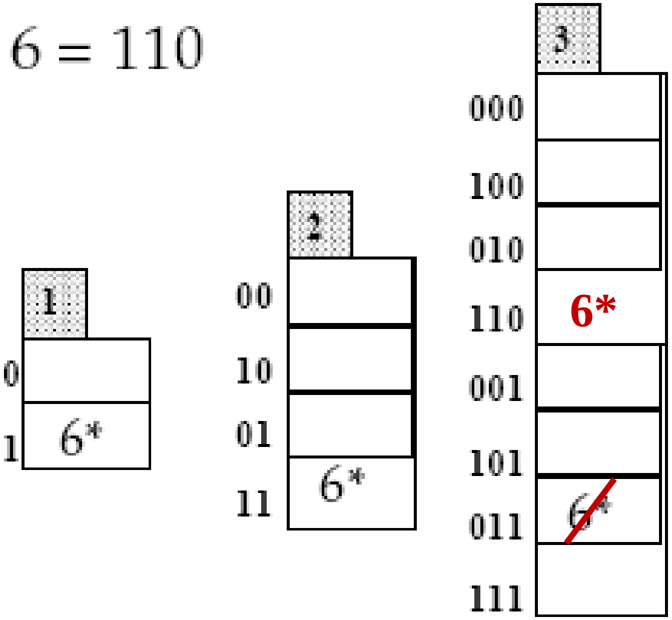
Extendible Hashing: Directory Doubling

- Using the **least significant bits** enables efficient doubling via copying of directory.



Least Significant

vs.



Most Significant

Extendible Hashing: Other Issues

- Efficiency:
  - If the directory fits in memory, an equality selection query can be answered with 1 disk I/O. Otherwise, 2 disk I/Os are needed.
- Deletions:
  - If removal of a data entry makes a bucket empty, then that bucket can be merged with its “split image”.
  - Merging buckets decreases the local depth.
  - If each directory element points to the same bucket as its split image, then we can halve the directory.

Linear Hashing: The Idea

- Linear Hashing handles the problem of long overflow chains without using a directory.
- **Idea:** Use a family of hash functions h_0, h_1, h_2, ..., such that
  - h_{i+1}'s range is twice that of h_i.
  - First, choose an initial hash function *h* and number of buckets *N*.
  - Then, $h_i(key) = h(key) \bmod (2^i N)$.
  - If $N = 2^{d0}$, for some *d0*, h_i consists of applying *h* and looking at the last *di* bits, where *di = d0 + i*.
  - Example: Assume *N* = 32 =2^5. Then:
    - *d0* = 5 (i.e., look at the last 5 bits)
    - $h_0 = h \bmod (1*32)$ (i.e., buckets in range 0 to 31)
    - *d1 = d0* + 1 = 5 + 1 = 6 (i.e., look at the last 6 bits)
    - $h_1 = h \bmod (2*32)$ (i.e., buckets in range 0 to 63)
    - … and so on.

Linear Hashing: Rounds of Splitting

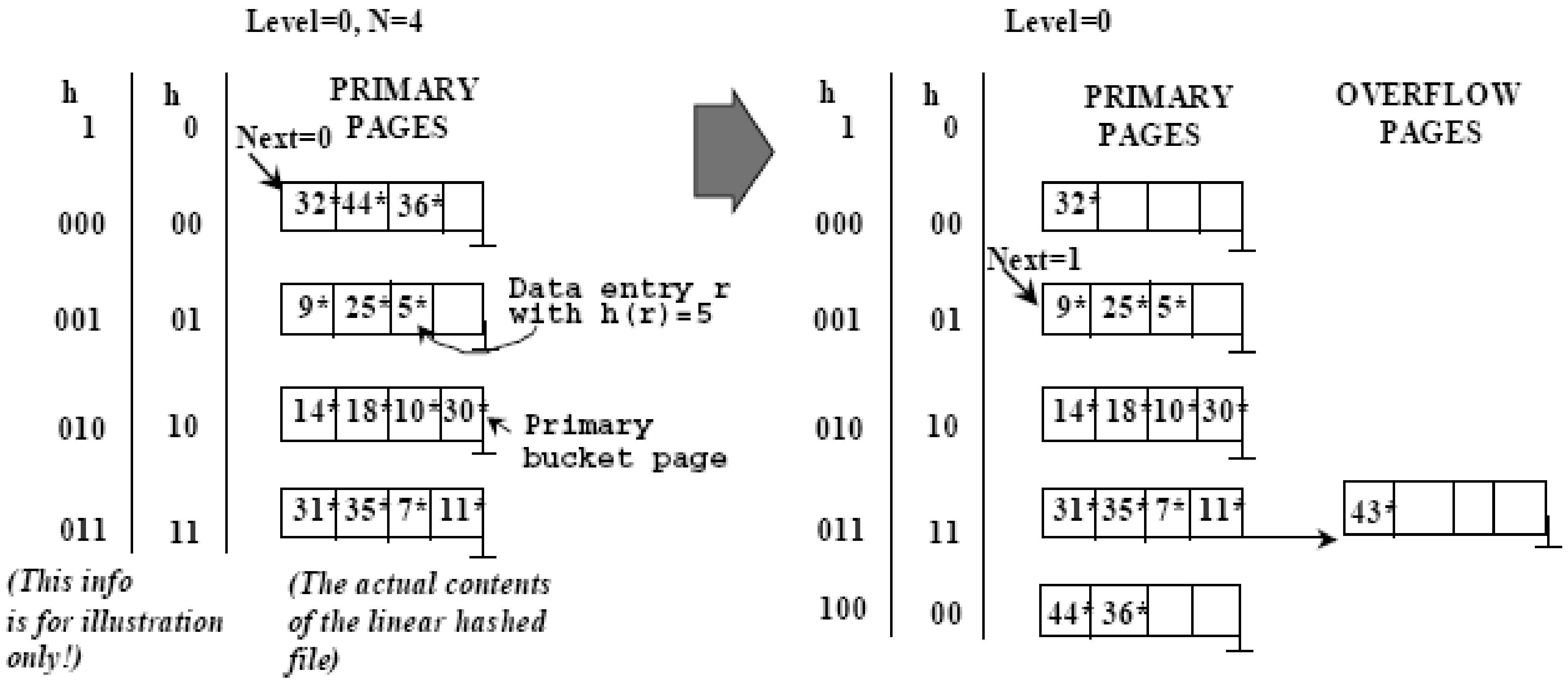
- Directory is avoided in Linear Hashing by using overflow pages, and choosing bucket to split in a round-robin fashion.
  - Splitting proceeds in “rounds”. A round ends when all N_R initial (for round *R*) buckets are split.
  - Current round number is “Level”. During the current round, only h_{Level} and $h_{Level+1}$ are in use.
  - Search: To find bucket for a data entry *r*, find $h_{Level}(r)$:
    - Assume: Buckets 0 to *Next-1* have been split; *Next* to N_R yet to be split.
    - If $h_{Level}(r)$ in range “*Next to* N_R”, *r* belongs here.
    - Else, *r* could belong to bucket $h_{Level}(r)$ or bucket $h_{Level}(r) + N_R$; must apply $h_{Level+1}(r)$ to find out.

Linear Hashing: Insertion

- Insertion: Find bucket by applying h_{Level} and $h_{Level+1}$:
  - If bucket to insert into is full:
    - Add overflow page and insert data entry.
    - Split *Next* bucket and increment *Next*.
- Since buckets are split round-robin, long overflow chains don't develop!
- Similar to directory doubling in Extendible Hashing.

Linear Hashing: An Example

- On split, $h_{Level+1}$ is used to re-distribute entries.



Summary of Hash-based Indexing

- Hash-based indexes are best for **equality selection queries**; they cannot support range selection queries.
- **Static Hashing** can lead to **long overflow chains**.
- **Dynamic Hashing**: Extendible or Linear.
  - **Extendible Hashing** avoids overflow pages by splitting a full bucket when a new data entry is to be added to it.
    - **Directory** to keep track of buckets, doubles periodically.
  - **Linear Hashing** avoids directory by splitting buckets **round-robin** and using **overflow pages**.
    - Overflow pages are not likely to be long (usually at most 2).

Indexing Recap

- Indexed Sequential Access Method (ISAM)
  - A **static**, tree-based index structure.
- B^+-trees
  - The database index structure; indexing based on any kind of (linear) **order**; adapts **dynamically** to inserts and deletes; low tree heights (~3-4) guarantee fast lookups.
- Clustered vs. Unclustered Indexes
  - An index is clustered if its underlying data pages are ordered according to the index; fast **sequential access** for clustered B^+-trees.
- Hash-Based Indexes
  - **Extendible hashing** and **linear hashing** adapt dynamically to the number of data entries.